CALCULATION OF A COLLISION BETWEEN METALLIC BODIES FROM A COMPRESSIBLE-LIQUID MODEL

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Pokrovskii [1] has given a description of the behavior of metals under the high dynamic loads arising in the collision of solids. Theoretical studies have been made [2, 3] on this, the interaction being considered on the basis of a model of ideal incompressible jets. Baum et al. [4] have examined collisions in which the compressibility and strength of the metals are taken into account.

Zlatin [5] has examined the effects of compressibility in the compression of metallic bodies. Sagomonyan [6] has proposed applying the methods of hypersonic gasdynamics to describe steady-state collision of solids. Here I consider steady-state collision of solids on the basis of two-dimensional flow with allowance for compressibility. Similar results have been obtained for one-dimensional flow [4, 5].

A metal rod (figure) of density ρ_0, whose length greatly exceeds its transverse dimensions, strikes a semi-infinite obstacle at right angles with a velocity V_0. Shock waves are produced in the rod and obstacle, and the material behind them is in a nearly liquid state. The liquid jet formed from the rod is turned through 180° by the high opposing pressure and acquires an axially symmetric mushroom form. The bounding surface AOA' between the rod and the obstacle is a contact surface, at which the pressure and the normal component of the velocity are continuous. Assuming that this surface can be approximated by a one-parameter surface (e.g., a sphere), we consider the case in which the speed U of that surface exceeds the speed of sound in the obstacle, while $V_0 - U$ is greater than the speed of sound in the rod. Then the shock waves in the rod and the obstacle are at rest relative to the contact surface. The main factor governing the entry of the rod into the obstacle is the pressure distribution over the contact surface, which will probably be similar to the pressure distribution for a body moving in an unbounded compressible liquid. This approach to the problem allows one to use the equations for flow of a compressible ideal liquid to describe the motion.

The following equations define the normal component u and tangential component v of the velocity vector W as well as the pressure p, density ρ, and entropy S in the variables r and θ (case of axial symmetry):

$$u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial \theta}-\frac{v^2}{r}+\frac{1}{\rho}\frac{\partial p}{\partial r}=0,$$
$$u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial \theta}+\frac{uv}{r}+\frac{1}{r\rho}\frac{\partial p}{\partial \theta}=0,$$
$$\frac{\partial u}{\partial r}+\frac{1}{r}\frac{\partial v}{\partial \theta}+\frac{v}{r\rho}\frac{\partial \rho}{\partial \theta}+\frac{u}{\rho}\frac{\partial \rho}{\partial r}+\frac{2u+v\operatorname{ctg}\theta}{r}=0,$$
$$u\frac{\partial S}{\partial r}+\frac{v}{r}\frac{\partial S}{\partial \theta}=0. \qquad (1)$$

The boundary conditions for system (1) are the conditions at the shock waves NN' and nn' (figure), whose meridional sections are given by $r_w(\theta)_1$ and $r_w(\theta)_2$, together with the conditions at the contact surface AOA', whose section is given by $r_k(\theta)$. The usual relations apply for the fluxes of mass, momentum, and energy at the shock waves:

$$\rho_0 v_{0n}=\rho v_n,\quad v_{0t}=v_t,\quad p_0+\rho_0 v_{0n}^2=p+\rho v_n^2,$$
$$h_0+{}^1/_2 v_{0n}^2=h+{}^1/_2 v_n^2\,. \qquad (2)$$

Here the subscript 0 denotes parameters of the unperturbed flow, while the subscripts n and t correspond to velocity components normal and tangential to the shock-wave front.

The pressure and the normal components of the velocity are continuous on AOA':

$$p_1=p_2,\quad u_1-v_1\frac{r_k'(\theta)}{r_k}=0,$$

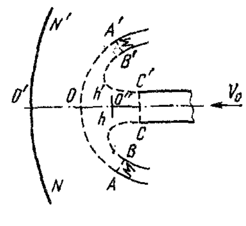
$$u_2-v_2\frac{r_k'(\theta)}{r_k}=0,\quad r_k'(\theta)=\frac{dr_k(\theta)}{d\theta}, \qquad (3)$$

where the subscript 1 refers to the obstacle and the subscript 2 to the rod.

The equation of state is taken in the following form [7]:

$$p=p_x(\rho)+\frac{3RT\gamma_p(\rho)\rho}{A}D\left(\frac{\theta}{T}\right)+\frac{\gamma_\theta}{2}\rho\beta(\rho)T^2,$$
$$E=E_x(\rho)+\frac{3RT}{A}D\left(\frac{\theta}{T}\right)+\frac{\beta(\rho)}{2}T^2.$$
$$p_x=\frac{1}{\rho^2}\frac{dE_x(\rho)}{d\rho}, \qquad (4)$$

in which T is absolute temperature, E is specific internal energy, E_x is the energy of the metal at 0° K (cold component of the energy), A is the atomic weight, R is the gas constant, $\gamma_p(\rho)$ is the Gruneisen



coefficient for the lattice, γ_θ is the Grüneisen coefficient for the electrons, β is the coefficient for the electron specific heat, θ is the Debye temperature, and $D(\theta/T)$ is the Debye function. Solid-state theory gives the following relation between $\gamma_p(\rho)$ and the cold component $p_x(\rho)$ of the pressure:

$$\gamma_p(v)=-\left(\frac{2}{3}-\frac{t}{3}\right)-\frac{v}{2}\frac{d^2}{dv^2}\left(p_x v^{2/3 t}\right)\left[\frac{d}{dv}\left(p_x v^{2/3 t}\right)\right]^{-1},$$

in which $v = 1/\rho$ is the specific volume.

The value t = 0 corresponds to the Landau-Stanyukovich-Slater theory [8, 9], while t = 1 corresponds to the Dugdale-MacDonald theory [10]. The ρ-dependence of p_x, E_x, $\gamma_p(\rho)$, and β is as follows:

$$p_x=Q\left[\delta^{2/3}\exp\{q(1-\delta^{-1/3})\}-\delta^{4/3}\right],$$
$$E_x=3\rho_{0k}Q\left[q^{-1}\exp\{q(1-\delta^{-1/3})\}-\delta^{1/3}\right],$$
$$\gamma_p=\frac{1}{6}\,\frac{q^2\delta^{-1/3}\exp\{q(1-\delta^{-1/3})\}-6}{\delta^{-1}q\exp\{q(1-\delta^{-1/3})\}-2},$$
$$\beta=\beta_0\delta^{\gamma_\theta},\quad \delta=\rho/\rho_{0k}, \qquad (5)$$

in which ρ_{0k} is the density of the metal at 0° K.

We use (4) to solve system (1) for region 1 (figure) subject to the boundary conditions of (2) by Telenin's method [11] for supersonic flow of a compressible fluid around a blunt body. The calculations were made for a copper rod meeting a copper half-space at speeds of 28.2, 22.6, 16.4, and 13.2 km/sec, which correspond, respectively, to pressures behind the shock wave in the obstacle of $9\cdot10^6$, $5\cdot10^6$, $2\cdot10^6$, and 10^6 atm. Other calculations were made for a lead rod and half-space meeting at speeds of 22.2, 15.8, and 9.2 km/sec, which correspond similarly to pressures of $9\cdot10^6$, $5\cdot10^6$, and 10^6 atm. All of the above parameters were derived for region 1.

The following is the dimensionless pressure along the zero ray from the contact surface ($\xi = 0$) to the shock wave ($\xi = 1$) for Cu and Pb at various V_0:

$\xi = 0.0$	0.2	0.4	0.6	0.8	1.0	Cu
$p = 0.7657$	0.7233	0.6597	0.5998	0.5321	0.4905	$(V_0=28.2)$
$p = 0.7364$	0.6712	0.5997	0.5379	0.4836	0.4326	$(V_0=22.6)$
$p = 0.6746$	0.5785	0.4973	0.4346	0.3832	0.3375	$(V_0=16.4)$
$p = 0.6245$	0.5072	0.4193	0.3563	0.3076	0.2659	$(V_0=13.2)$

$\xi = 0.0$	0.2	0.4	0.6	0.8	1.0	Pb
$p = 0.8101$	0.7936	0.7700	0.7192	0.6652	0.6076	$(V_0=22.2)$
$p = 0.7872$	0.7743	0.7228	0.6675	0.6130	0.5577	$(V_0=15.8)$
$p = 0.7049$	0.6154	0.5324	0.4662	0.4109	0.3617	$(V_0=9.2)$

The distances of the shock wave in region 1 from the contact surface are much greater for the metals than for a real gas at the same Mach number, which may be due to the much smaller density change at the shock-wave front in a metal.

Calculations for region 2 were performed on the basis of the conservation of mass and momentum. The jet from the rod flows away along A0A', on which surface we know the distributions of the velocity pressure, and density from the solution for region 1. The following are the conservation equations for the volume ABCC'B'A'0, which is indicated by the dashed line in the figure:

$$\int_{\Sigma} \rho w_n \, ds = \frac{1}{2}\pi\rho_0 V_0 r_0^2, \quad \int_{\Sigma} \rho w_n w \, ds - \pi\rho_0 V_0^2 r_0^2 =$$

$$= \int_{\Sigma'} (p - p_0)\, n \, ds \quad (\Sigma' = \Sigma + \Sigma_{AOA'}). \tag{6}$$

We can solve (6) if we know the pressure distribution along A0A' and the exact values for the pressure, density, and velocity at A and A', provided some assumption is made about the distribution of the flow parameters in section Σ (figure). We assume that the local velocity of sound is reached at n and n' (where the shock waves reaches the free surface). Then the relationships at the shock-wave front allow us to determine the angle of inclination φ of the shock wave at n and n' to the positive direction of the axis 00'. From the known angle of the step at n and n' we deduce the pressure at these points. Bernoulli's integral applies along the free surface nB and n'B', while the pressure at B and B' is known, being simply p_0; hence, the velocity and density at B and B' may be determined. Let p_A, ρ_A, w_A and p_B, ρ_B, w_B be the pressure, density, and velocity, respectively, at A and B. If we approximate the parameters in section Σ by linear functions of the distance from the contact surface, we have

$$p = \frac{p_A - p_B}{r_A - r_B}(r - r_B) + p_B,$$

$$\rho = \frac{\rho_A - \rho_B}{r_A - r_B}(r - r_B) + \rho_B,$$

$$w = \frac{w_A - w_B}{r_A - r_B}(r - r_B) + w_B. \tag{7}$$

Substitution of (7) into (6) gives two algebraic equations for r_A and r_B as functions of the radius r_0 of the rod before collision. When a copper rod meets a copper half-space at 28.2 km/sec, r_A is $2.52r_0$, while the diameter of the jet in the section Σ is $r_A - r_B = 0.55r_0$.

Analogous calculations have been performed for all the cases of collision.

REFERENCES

1. G. I. Pokrovskii, Hydrodynamics of High Speeds [in Russian], Izd. Znanie, 1966.

2. M. A. Lavent'ev, "Principles of operation of a shaped charge," Uspekhi matem. nauk, vol. 12, no. 4, p. 41, 1957.

3. G. F. Birkhoff, D. P. McDougall, E. M. Pugh, and G. J. Taylor, "Explosives with lined cavities," J. Appl. Phys., vol. 19, 563, 1949.

4. F. A. Baum, K. P. Stanyukovich, and B. I. Shekhter, The Physics of Explosions [in Russian], Fizmatgiz, 1959.

5. N. A. Zlatin, "Theory of high-speed collisions of metallic bodies," Zh. Tekh. Fiz., vol. 31, no. 8, p. 911, 1961.

6. A. Ya. Sagomonyan, "Interaction of bodies at high speeds," Dokl. AN SSSR, vol. 156, no. 5, p. 1053, 1964.

7. L. V. Al'tshuler, A. A. Bakanova, and R. F. Trunin, "Shock adiabats and zero isotherms for seven metals at high pressures," Zh. eksp. i teor. fiz., vol. 42, no. 1, p. 91, 1962.

8. L. D. Landau and K. P. Stanyukovich, "Study of the detonation of condensed explosives," Dokl. AN SSSR, 46, no. 9, 399, 1945.

9. I. C. Slater, Introduction to Chemical Physics, New York, 1939.

10. J. S. Dugdale and D. K. MacDonald, "The thermal expansion of solids," Phys. Rev., vol. 89, no. 4, 832, 1957.

11. S. M. Gilinskii, G. F. Telenin, and G. P. Tinyakov, "Methods of of calculating supersonic flow around a blunt body with a detached shock wave," Izv. AN SSSR, Mekhanika i Mashinostroenie, no. 4, p. 8, 1964.

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